| Constellation number | Almagest celestial area | Latin name of the constellation | Percentage of poorly-identifiable stars |  |  | Number of stars |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | In a "pure" constellation | In a constellation with informata | In the informata | In a "pure" constellation | In the informata |
| 1 | A | Ursa Minor | 0 | 0 | 0 | 7 | 1 |
| 2 | A | Ursa Major | 3.7 | 11.4 | 38 | 27 | 8 |
| 3 | A | Draco | 0 | - | - | 31 | 0 |
| 4 | A | Cepheus | 0 | 7.7 | 50 | 11 | 2 |
| 5 | A | Bootes | 27.3 | 26 | 0 | 22 | 2 |
| 6 | A | Corona Boreal. | 0 | - | - | 8 | 0 |
| 7 | A | Hercules | 10.3 | 10 | 0 | 29 | 1 |
| 8 | A | Lyra | 10 |  | - | 10 | 0 |
| 9 | M | Cygnus | 0 | 0 | 0 | 17 | 2 |
| 10 | M | Cassiopeia | 23 | - | - | 13 | 0 |
| 11 | M | Perseus | 3.8 | 6.9 | 33.3 | 26 | 3 |
| 12 | A, M | Auriga | 21.4 | - | - | 14 | 0 |
| 13 | M | Ophiuchus | 25 | 20.7 | 0 | 24 | 5 |
| 14 | M | Serpens | 0 | - | - | 18 | 0 |
| 15 | M | Sagitta | 0 | - | - | 5 | 0 |
| 16 | B | Aquila | 22.3 | 13.3 | 0 | 9 | 6 |
| 17 | B | Delphinus | 20 | - | - | 10 | 0 |
| 18 | B | Equuleus | 100 | - | - | 4 | 0 |
| 19 | B | Pegasus | 10 | - | - | 20 | 0 |
| 20 | B | Andromeda | 13 | - | - | 23 | 0 |
| 21 | B | Triangulum | 0 | - | - | 4 | 0 |
| 22 | ZodB | Aries | 0 | 0 | 0 | 13 | 5 |
| 23 | ZodB | Taurus | 21.2 | 25 | 36.4 | 33 | 11 |
| 24 | ZodA | Gemini | 5.6 | 20 | 57 | 18 | 7 |
| 25 | ZodA | Cancer | 0 | 23 | 75 | 9 | 4 |
| 26 | ZodA | Leo | 11.1 | 17.1 | 37.5 | 27 | 8 |
| 27 | ZodA | Virgo | 15.4 | 15.6 | 16.6 | 26 | 6 |
| 28 | ZodA | Libra | 0 | 23.5 | 44.4 | 8 | 9 |
| 29 | ZodA | Scorpius | 4.8 | 12.5 | 66.7 | 21 | 3 |
| 30 | ZodB | Sagittarius | 12.9 | - | - | 31 | 0 |
| 31 | ZodB | Capricornus | 3.6 | - | - | 28 | 0 |
| 32 | ZodB | Aquarius | 26.1 | 24.4 | 0 | 42 | 3 |
| 33 | ZodB | Pisces | 5.8 | 5.2 | 0 | 34 | 4 |
| 34 | D | Cetus | 22.7 | - | - | 22 | 0 |
| 35 | D | Orion | 8.9 | - | - | 38 | 0 |
| 36 | D | Eridanus | 26.4 | - | - | 34 | 0 |
| 37 | D | Lepus | 0 | - | - | 12 | 0 |
| 38 | D | Canis Major | 5.6 | 41.3 | 100 | 18 | 11 |
| 39 | C | Canis Minor | 0 | - | - | 2 | 0 |
| 40 | C | Argo Navis | 68.9 | - | - | 45 | 0 |
| 41 | C | Hydra | 16 | 22.2 | 100 | 25 | 2 |
| 42 | C | Crater | 57.1 | - | - | 7 | 0 |
| 43 | C | Corvus | 0 | - | - | 7 | 0 |
| 44 | C | Centaurus | 81 | - | - | 37 | 0 |
| 45 | C | Lupus | 100 | - | - | 19 | 0 |
| 46 | C | Ara | 100 | - | - | 7 | 0 |
| 47 | D | Corona Austr. | 100 | - | - | 13 | 0 |
| 48 | D | Pisces Austr. | 8.3 | 38.9 | 100 | 12 | 6 |

Table 2.2. Percentage of poorly identifiable stars in Almagest constellations.

- region $B$ : constellations 16-23 and 30-33;
- region $\operatorname{Zod} A$, which is part of region $A$ : constellations 24-29;
- region $\operatorname{Zod} B$, which is part of region B: constellations 22, 23, 30-33;
- region D: constellations 34-38, 47 and 48.
- region C: constellations 39-46;
- region M: constellations 9-15.

Corollary 2. The stars that constitute the informata in the Almagest were measured with comparatively low precision, with the exception of the following: 1 star in Ursa Minor, 1 star in Boötes, 1 star in Hercules, 2 stars in Cygnus, 5 stars in Ophiuchus, 6 stars in Aquila, 5 stars in Aries, 3 stars in Aquarius and 4 stars in Pisces, or 9 informata out of the total of 22.

The remaining thirteen informata were measured very badly. Indeed, we find $38 \%$ of poorly measured stars in the informata of Ursa Major, $50 \%$ in the informata of Cepheus, $33.3 \%$ in the informata of Perseus, $36.4 \%$ in the informata of Taurus, $57 \%$ in the informata of Gemini, $75 \%$ in the informata of Cancer, $37.5 \%$ in the informata of Leo, $16.6 \%$ in the informata of Virgo, $44.4 \%$ in the informata of Libra, $66.7 \%$ in the informata of Scorpio, and $100 \%$ in the informata of Canis Major, Hydra and Piscis Austrinus.

And so, there are lots of poorly measured stars in the informata of the Almagest in general. It would be apropos to voice the hypothesis (one that doesn't affect our further research in any way at all, as a matter of fact) that the stars collected in the informata did not constitute the primary "constellation pattern", which is why the measurement of their coordinates was performed with less precision - especially if the star in question was a dim one. Of course, if a bright star ended up among the informata, its coordinates could be measured with greater diligence. For instance, the famous Arcturus is part of the well-measured informata of Aquarius. However, table 2.2 shows us that in a typical situation the stars of the informata are measured with less precision than the stars of the "pure" constellation.

It would therefore strike one as natural to separate the informata from the main stars of the constellation for the time being. Actually, this is how it is done in the Almagest - the informata stars are gathered in a separate eponymous group. We shall consider the "pure" constellations alone.

This is the very reason why we introduced two separate columns in table 2.2 - one corresponds to the share of poorly identifiable stars in the "pure" constellation, and the other - to the main stars of the constellation with the informata added thereto. Our analysis of the fourth column demonstrates the picture to be completely different here. Apart from the "pure" constellations that were measured with relatively high accuracy, there are some whose stellar coordinates are less accurate.

For greater demonstrability, we have transcribed the numeric data from the fourth and the fifth column in the following manner:

Inside each of the constellations reproduced as a certain area confined within a zigzagging border there are two numbers. The fraction's nominator represents the share of poorly measured stars in the current "pure" constellation, sans the informata. The fraction's denominator contains the percentage of poorly measured stars together with the informata. There is no denominator if the constellation in question contains no informata; however, the fraction line is nonetheless present. The dotted line one sees in fig. 2.15 represents the Milky Way.

In order to facilitate the analysis of the above picture, let us count the average share of poorly identifiable stars separately (for each of the seven regions as described above). We shall add up the previously calculated rates for every constellation and divide the result by the number of constellations in the region. The result is represented in table 2.3.

Let us turn to fig. 2.16, where different regions are represented by different kinds of shading. They correspond to varying levels of observation quality. White colour stands for values between $0 \%$ and $5 \%$ of poorly measured stars. Dotted shading represents values falling between $6 \%$ and $10 \%$, slanted shading - values between $21 \%$ and $30 \%$, and, finally, black field stands for values between $31 \%$ and $100 \%$ of stars whose coordinates lack precision.

Thus, the darker a given area, the worse the quality of its measurement in the Almagest. We instantly notice the fact that many austral constellations in Area $C$, to the right of the Milky Way, are measured very poorly indeed - we see a lot of solid black shading here, qv in fig. 2.16. On the other hand, the constellations in Area $A$ are measured a great deal better,


Fig. 2.15. Inside each of the constellations mentioned by Ptolemy and drawn as an area with zigzagged boundaries we specify two numbers, the first one corresponding to the percentage of poorly-measured stars in a constellation without informata, and the lower - to the same in a constellation with the informata added.


Fig. 2.16. A demonstrable representation of well-measured and poorly-measured celestial areas from the Almagest. The darker the area, the less accurate the corresponding measurements.
there is a lot of white here. Area $B$, which lays to the left of Area $M$, is measured worse than Area $A$, we see a good deal of double shading. Some of the areas in fig. 2.16 are marked with a question mark - they are the regions of the modern celestial sphere that formally remain beyond the confines of the Almagest constellations. Seeing as how the Almagest gives no
precise definitions of constellation borders, neighbouring constellations may become "stretched" in such a way that they will fill the empty zones in fig. 2.16. We shall refrain from describing this procedure in greater detail - there are few such "blank spots", and they hardly influence our results in any way at all.

For a more illustrative analysis of the above picture, let us calculate the average percentage of poorly identifiable stars in each of the above seven areas individually by adding up the percentages calculated above for each of the constellations and dividing the sub by the total number of constellations for each area. The result is represented in table 2.3.

Corollary 3. Region $A$ is measured better than regions $B, C, D$ and $M$ in the Almagest - namely, $6.3 \%$ of poorly identifiable stars in "pure" constellations and $12.6 \%$ in constellations with added informata.

Corollary 4. Region $B$ is measured worse than region $A$ in the Almagest, namely, we have $19.6 \%$ of poorly identifiable stars in the "pure" constellations and $19 \%$ in the constellations with the informata.

Corollary 5. Region $M$, or the Milky Way, occupies an intermediate position between regions $A$ and $B$ - $10.5 \%$ of poorly identifiable stars in "pure" constellations and $10.3 \%$ in the constellations with informata.

Corollary 6. Regions $C$ and $D$ are measured the worst in the Almagest - namely, region $D$ contains $27.4 \%$ of poorly identifiable stars in "pure" constellations and $36.9 \%$ in constellations with informata added. For region $C$ the percentage of poorly identifiable stars equals $52.9 \%$ in "pure" constellations and $53.6 \%$ in constellations with informata.

| Parts of the celestial sphere <br> in the Almagest | $A$ | B | $A w / o$ <br> ZodA | B w/o <br> ZodB | ZodA | ZodB | $D$ | $C$ | $M$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of constellations | 14 | 12 | 8 | 6 | 6 | 6 | 7 | 8 | 7 |
| Constellation numbers in the Almagest | $1-8$, <br> $24-29$ | $16-23$, <br> $30-33$ | $1-8$ | $16-21$ | $24-29$ | 22,23, <br> $30-33$ | $34-38$, <br> 47,48 | $39-46$ | $9-15$ |
| Percentage of poorly identifiable stars <br> in "pure" constellations (w/o informata) | 6,3 | 19,6 | 6,4 | 27,6 | 6,2 | 11,6 | 27,4 | 52,9 | 10,5 |
| Percentage of poorly identifiable stars <br> in constellations with informata | 12,6 | 19 | 8,1 | 26,5 | 18,6 | 11,9 | 36,9 | 53,6 | 10,3 |
| Percentage of reliably identifiable stars <br> in "pure" constellations | 93,7 | 80,4 | 93,6 | 72,4 | 93,8 | 88,4 | 72,6 | 47,1 | 89,5 |

Table 2.3. Average percentage of poorly identifiable stars as given for each of the seven areas individually.

Corollary 7. Region Zod A is measured best in the Almagest - it is the part of the Zodiac on the right of the Milky Way. It includes the constellations of Gemini, Cancer, Leo, Virgo and Scorpio. Here we have a mere $6.2 \%$ of poorly identifiable stars in "pure" constellations.

Corollary 8. Region Zod B is measured much worse than Zod A. Here we have 11.6\% of poorly identifiable stars in "pure" constellations. Region Zod B comprises the constellations of Sagittarius, Capricorn, Aquarius, Pisces, Aries and Taurus.

In order to get a better idea of what the information in table 2.3 really stands for, we have drawn a diagram, which is reproduced in fig. 2.14. Different kinds of shading correspond to different levels of measurement precision, or the percentage of dubiously identified stars. The white zone stands for areas that contain $0 \%$ to $10 \%$ of such stars, dotted shading corresponds to levels of $10 \%-20 \%$, linear shading - to those of $20 \%-30 \%$, and double shading represents zones of the celestial sphere that contain 30\% to $100 \%$ of stars whose identity is ambiguous.

Another illustrative representation of the above information can be seen in fig. 2.17. The numbers of all 48 Almagest constellations are placed horizontally in such a way that they form groups, such as $A, B$, $\operatorname{Zod} A, \operatorname{Zod} B, A-\operatorname{Zod} A$ ( $A$ without $\operatorname{Zod} A$, that is), $B-\operatorname{Zod} B, C, D$ and $M$. The respective percentage of dubiously identified stars in "pure" constellations is aligned vertically. Each of the constellation groups as listed above is represented by a certain horizontal segment in fig. 2.17 - the average percentage value for the group under consideration. Fig. 2.17 makes it perfectly obvious that the coordinates of stars in "group $A$ " were measured with maximum precision (regions $A, \operatorname{Zod} A$ and $A-\operatorname{Zod} A$ ). Corresponding values are the smallest. "Group $B$ " is located much further up in fig. 2.17, which stands for lower measurement precision in this area. It is also apparent that the stars of the Southern Hemisphere were measured even worse.

The same information can be found in fig. 2.18, which is based on the last line of table 2.3 , where the dubiously identified star percentage values in "pure" Almagest constellations are aligned vertically. This graph is obviously implied by the graph in fig. 2.17 and represents the values of the latter subtracted from $100 \%$.


Fig. 2.17. Percentage of dubiously identified stars in the "pure" constellations of the Almagest, without accounting for the stars listed in the informatae. It is quite obvious that the stars from "group $A$ " were measured the best, and the percentage of dubious stars here is the lowest.

Corollary 9. The first primary statement. The seven regions of the Almagest star atlas that we have discovered differ by the precision of stellar coordinate measurements. Indeed, different kinds of shading correspond to the seven celestial regions as described above ( $A, B, C, D, M, \operatorname{Zod} A$ and $\operatorname{Zod} B$ ) in fig. 2.14.

Corollary 10. The second primary statement.

1) Further research of star coordinates in the Almagest has to be based on the stars from region $A$ first and foremost, since it is the most accurately measured region with a minimum of dubiously identified stars.
2) One mustn't base any corollaries on the study of the stars from regions $C$ and $D$. An exceptionally large number of poorly identifiable stars in this area tells us quite explicitly that the regions in question cannot be considered reliably measured. Refraction is one of the reasons why the southern stars could not be measured with sufficient precision by the author of the Almagest - it is common knowledge that the coordinates of the stars located close to the horizon are affected by light refraction.
3) We get the opportunity to differentiate the list of 12 named stars by the level of their "reliability". The
stars measured with the greatest accuracy correspond to region $A$ and its immediate vicinity. They are Regulus, Spica, Previndemiatrix, Procyon, Arcturus, Acelli, Antares, Lyra (Vega), and Capella. The "ambiguous" stars are Sirius (region $D$ ), Aquila, or Altair - region B, left border of the Milky Way, and Canopus, which is altogether off the chart. These stars ended up in the "poorly measured" celestial regions.

Incidentally, the star Previndemiatrix also has to be excluded from the list of "good" named stars for the following reason. Although this star can be identified quite well (in particular, it is absent from the list of poorly identifiable stars, qv in table 6 in [1339]), its coordinates as given in [1339] are rather uncertain and not


Fig. 2.18. Percentage of reliably identified stars in the "pure" constellations of the Almagest. substantiated with any references to the original Almagest manuscripts. Peters reports the following about the coordinates of the star Previndemiatrix in the Almagest: "Greek sources indicate $20^{\circ} 10^{\prime}$, and the Arabs - $15^{\circ} 10^{\prime}$ [a discrepancy of five degrees, no less - Auth.]. Ulugbek's catalogue contains the coordinates of $16^{\circ} 15^{\prime}$. Peters states $16^{\circ} 0^{\prime}$, following the catalogue of Halma, likewise Bailey however, he points out that Halma gives no authoritative references. It is clear that Halma's $16^{\circ} 0^{\prime}$ were taken from Halley, which is correct [?! - Auth.] but not supported by any manuscripts" ([1339], page 104). It is clear that a situation as ambiguous as this one requires the star Previndemiatrix to be excluded from further consideration.

Thus, eight out of twelve named stars of the Almagest end up in the "reliably measured" region of the celestial sphere: Regulus, Spica, Procyon, Arcturus, Acelli, Antares, Lyra (Vega), and Cappella.

## 4.

## POSSIBLE DISTORTION OF THE STAR COORDINATES RESULTING FROM THE ATMOSPHERIC REFRACTION

A researcher of a star catalogue must always remember the physical phenomenon of refraction, whose influence can greatly distort the coordinates of the southern stars.

The phenomenon of refraction owes its existence to the properties of the atmosphere that affect the measurements conducted from the surface of the Earth; the latter is the case with all the ancient observations. From the mathematical point of view, the atmosphere of the Earth can be regarded as a set of concentric spherical air layers whose density is more or less uniform, changing from layer to layer.

It is common knowledge that a ray of sunshine is subject to refraction as it moves between different atmospheric layers of different density (see fig. 2.19). The ray becomes more vertical as a result, approximating the normal, which is the perpendicular border of two layers.

In fig. 2.20 we see a diagram of the Earth's atmosphere, presented as a set of concentric layers whose density diminishes as altitude grows. A ray of light that comes from star $A$ refracts as it moves from one layer to another. As a result, it moves through the atmosphere forming a certain curve that can be calculated from the corresponding equation. This was done in the theory of atmospheric refraction. The result is shown in fig. 2.20 - the observer located in point $O$ on the surface of the Earth perceives star $B$ as part of half-line $O B$, while in reality the direction is represented by half-line $O A^{\prime}$. Therefore, refraction "lifts" stars in a certain way.

